

Exhibit 3

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Economic valuation of product features

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Abstract We develop a market-based paradigm to value the enhancement or addition of features to a product. We define the market value of a product or feature enhancement as the change in the equilibrium profits that would prevail with and without the enhancement. In order to compute changes in equilibrium profits, a valid demand system must be constructed to value the feature. The demand system must be supplemented by information on competitive offerings and cost. In many situations, demand data is either not available or not informative with respect to demand for a product feature. Conjoint methods can be used to construct the demand system via a set of designed survey-based experiments. We illustrate our methods using data on the demand for digital cameras and demonstrate how the profits-based metric provides very different answers than the standard welfare or Willingness-To-Pay calculations.

Keywords Product features · Conjoint · Equilibrium profits · Bayesian analysis

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1 Introduction

Valuation of product features is a critical part of the development and marketing of products and services.¹ Firms continuously seek to improve existing products by adding new features. Many “new products” are essentially old products which have been enhanced with features previously unavailable. For example, consider the smartphone category of products. As new generations of smartphones are produced and marketed, existing features such as screen resolution/size or cellular network speed are enhanced to new higher levels. In addition, features are added to enhance the usability of smartphone. These new features might include integration of social networking functions into the camera application of the smartphone or increasing battery life. New and enhanced features often involve substantial development costs and sometimes also require new components which drive up the marginal cost of production.

The decision to develop new features is a strategic decision involving not only the cost and revenue potential of adding the feature but also the possible competitive response. The development and marketing costs of feature enhancement must be weighed against the expected increase in profits which will accrue if the product feature is added or enhanced. Expected profits in the marketplace in which the firm competes using the enhanced product should be compared to expected profits in the marketplace in which the firm’s product is not enhanced. Computing this change in expected profits involves predicting not only demand for the feature but also assessing the new industry equilibrium that will prevail with a new set of products and competitive offerings.

To undertake equilibrium computations, we must build a valid demand system, obtain cost information, and add assumptions regarding the nature of competition. For many product features, informative demand data are difficult to come by. For example, if a product feature is genuinely new to the market, there is no existing data that can be used to estimate the demand for this product characteristic. In other situations where features have been introduced into the market, we do not see sufficient price variation or we may have concerns regarding the endogeneity of prices due to the standard omitted characteristics argument (see, for example, Berry et al. (1995)). Conjoint-based surveys can be thought of as an experimental approach to assessing demand for products. As has long been recognized, the virtue of conjoint analysis is that product features can be experimentally manipulated to tease out the demand for specific product features. Data obtained from randomized experiments or simulated markets do not suffer from the endogeneity and other measurement concerns that sometimes compromise market data. All of the variation in price and characteristics in conjoint data can be used to estimate data, instead of a smaller fraction identified through the use of instruments or other measures.

¹Valuation of product features is also critical in patent disputes. In particular, damages from patent infringement are typically assessed as either royalty payments or lost profits. In either case, a valuation of the patented feature in terms of incremental profits should be at the heart of any valid damages calculation. In a companion paper, we consider the problem of patent damage calculations and provide explicit methods for using conjoint data to undertake these calculations (Allenby et al. 2014).

While conjoint analysis has long been appreciated as a valuable tool for estimating consumer preferences, the use of conjoint data for equilibrium calculations has seldom been considered. Valuation of product features is typically conducted via market share simulations and Willingness To Pay (hereafter WTP) calculations. Obviously, these calculations do not directly relate to the value of the product feature to the firm. The only sensible metric for assessing firm value is profit. In calculating changes in profits that can be ascribed to the product feature, competitive response must be considered, necessitating some sort of equilibrium computation. To compute equilibrium outcomes, we will have to make assumptions about cost, the nature of competition, and the set of competitive offers. Conjoint studies will have to be designed with this in mind. In particular, greater care to include an appropriate set of competitive brands, handle the outside option appropriately, and estimate price sensitivity precisely must be exercised.

Our work should be distinguished from the work on the valuation of new products (see, for example, Trajtenberg (1989) and Petrin (2002)) in two critical aspects. First, the emphasis of the economics literature on new products is, not surprisingly, on the welfare effects of the new product introduction, while our emphasis is on the profit consequences for the firms adding a new product feature. Second, we do not add a new logit error for the products with feature enhancement as is common in the new product literature. It is well known that the infinite support of the logit error creates consumer surplus even for products which are dominated on observable features. Since we focus on feature enhancement, we are able to observe exactly what the “new product” is comprised of and do not have to assume there are unobservable aspects of the new product which create value through horizontal differentiation.

In the marketing literature, Ofek and Srinivasan (2002) have considered the problem of what they term “the market value of an attribute improvement (MVAI).” They define the MVAI for a feature enhancement as the increase in price that can be made to an enhanced product while keeping its share constant (p. 401 see text above equation (9)). This is similar to the “market simulation” approach to computing WTP in the conjoint literature. However, the MVAI measure does not consider the competitive response of other firms and is based exclusively on conjoint-derived demand parameters without any consideration of costs or a competitive equilibrium. We believe that the economic value of a feature enhancement must be based on what profits a firm can earn on the enhanced product which is not measured by MVAI.

There have been efforts to compute Nash equilibrium prices using characteristics models in the marketing literature. Horsky and Nelson (1992) use a cross-sectional survey on car purchases and preferences to compute Nash price equilibrium. However, their approach does not specify a valid demand system constructed from first principles and does not allow for a full distribution of heterogeneous preferences. Choi and DeSarbo (1994) consider the problem of product design and pricing and employ a branch-and-bound algorithm to search for equilibrium points. The discrete nature of product positioning decisions makes it difficult to search for equilibria and their model does not allow for heterogeneous consumers. Srinivasan et al. (1997) discuss Nash pricing equilibrium but do not compute equilibria using a valid demand model with heterogeneous consumers. None of these papers are able to provide full

finite-sample inference for either equilibrium quantities or firm profits as we demonstrate below. In addition, these papers do not consider the problem of valuation of a product feature or use equilibrium incremental profits as a metric for feature valuation.

We illustrate our profit-based approach to valuing product features using the digital camera survey. We designed and fielded a conjoint survey for the purpose of undertaking equilibrium calculations to value a particular camera feature. We contrast our equilibrium results to purely demand-based measures such as WTP and show that there are economically large differences between these approaches.

2 Economic valuation of features

The goal of feature enhancement is to improve profitability of the firm by introducing a product with feature enhancement into an existing market. For this reason, we believe that the only sensible measure of the economic value of feature enhancement is the incremental profits that the feature enhancement will generate.

$$\Delta\pi = \pi(p^{eq}, m^{eq}|A^*) - \pi(p^{eq}, m^{eq}|A) \quad (2.1)$$

π is the profits associated with the industry equilibrium prices and shares given a particular set of competing products which is represented by the choice set defined by the attribute matrix. A^* denotes the set of products where one of the products has been enhanced by adding or improving a product feature. A represents the set of products without feature enhancement. The set of products in the market is defined via their vector of characteristics.

The equilibrium depends on the set of products offered in the market place. (p^{eq}, m^{eq}) is the outcome of a price equilibrium with m denoting the vector of market shares. An equilibrium is defined as a set of prices and accompanying market shares which satisfy the conditions specified by a particular equilibrium concept. We use the standard Nash Equilibrium concept for differentiated products. While we concentrate on a pricing-game equilibrium, our idea that valuation should be obtained from long-run equilibrium profit consequences can be applied to more complicated multi-stage games where there are equilibria computed in the choice of both features and price. Our view is that the price equilibrium is the logical starting point and that for the question of the marginal value of a feature it is the pricing game that determines the value of the feature. In the patent setting, there are restrictions on whether or not other firms can add the feature (if the feature is enabled by a patent) and it would not be appropriate to consider an equilibrium involving feature choice.

In the abstract, our definition of economic value of feature enhancement seems to be the only relevant measure for the firm that seeks to enhance a feature. All funds have an opportunity cost and the incremental profits calculation is fundamental to deploying product development resources optimally. In fairness, industry practitioners of conjoint analysis also appreciate some of the benefits of an incremental profits orientation. Often, marketing research firms construct “market simulators” that simulate market shares given a specific set of products in the market. Some even go further as to attempt to compute the “optimal” price by simulating different market

shares corresponding to different “pricing scenarios.” In these exercises, practitioners fix competing prices at a set of prices that may include their informal estimate of competitor response. This is not the same as computing a marketing equilibrium but moves in that direction.

2.1 Assumptions

Once the principle of incremental profits is adopted, the problem becomes one of defining the nature of competition, the competitive set and to choose an equilibrium concept. These assumptions must be added to the assumptions of a specific parametric demand system (we will use a heterogeneous logit demand system which is flexible but still parametric) as well as the assumption (implicit in all conjoint analysis) that products can be well described by bundles of attributes. Added to these assumptions, our valuation method will also require cost information.

Specifically, we will assume

1. Demand Specification: A standard heterogeneous logit demand that is linear in the attributes (including price)
2. Cost Specification: Constant marginal cost²
3. Single product firms
4. Feature Exclusivity: The feature can only be added to one product
5. No Entry-Exit: Firms cannot enter or exit the market after product enhancement takes place
6. Static Nash Price Competition

Assumptions 2, 3, 4 can be easily relaxed. Assumption 1 can be replaced by any valid or integrable demand system. Assumptions 5 and 6 cannot be relaxed without imparting considerable additional complexity to the equilibrium computations.

2.2 A standard logit model for demand

Valuation of product features depends on a model for product demand. In most marketing and litigation contexts, a model of demand for differentiated products is appropriate. We briefly review the standard choice model for differentiated product demand. In many contexts, any one customer purchases at most a single unit of the product. While it is straightforward to extend our framework to consider the quantity decision, we limit attention to the unit demand situation. The demand system then becomes a choice problem in which customers have J choice alternatives, each with characteristics vector, a_j , and price, p_j . The standard random utility model (McFadden 1981) postulates that the utility for the j th alternative consists of a deter-

²In equilibrium after a feature introduction, it is possible that there may be quantity adjustments which may change the marginal cost of production. It would be a simple matter to add a specification in which cost changes as a function of quantity produced. It should be noted that this is yet another way in which the failure to account for equilibrium adjustments may bias traditional approaches to feature valuation such as Willingness To Pay.

ministic portion (driven by a and p) and an unobservable portion which is modeled, for convenience, as a Type I extreme value distribution.

$$u_j = \beta' a_j - \beta_p p_j + \varepsilon_j \quad (2.2)$$

a_j is a $k \times 1$ vector of attributes of the product, including the feature that requires valuation, one of which is the focal feature.

In choice models used for demand data, it is now common to include an aggregate demand shock, *à la* Berry et al. (1995). In the conjoint design, we explicitly delineate the relevant set of characteristics and instruct respondents to assume that all other characteristics are held constant across choice alternatives. This breaks any possible correlations between observed and unobserved characteristics.

Feature enhancement is modeled as alternative levels of the focal feature, a_f (one element of the vector a), while additional features would simply have a_f as a dummy or indicator variable. There are three important assumptions regarding the model in Eq. 2.2 that are important for feature valuation: 1. this is a compensatory model with a linear utility, 2. we enter price linearly into the model instead of using the more common dummy variable coding used in the conjoint literature,³ 3. there is a random utility error with infinite support. The random utility error, ε_j , represents the unobservable (to the investigator) part of utility. This means that actual utility received from any given choice alternative depends not only on the observed product attributes, a , and price but also on realizations from the error distribution. In the standard random utility model, there is the possibility of receiving up to infinite utility from the choice alternative. This means that in evaluating the option to make choices from a set of products, we must consider the contribution not only of the observed or deterministic portion of utility but also the distribution of the utility errors. The possibilities for realization from the error distribution provide a source of utility for each choice alternative.

The random utility model was designed for application to revealed preference or actual choice in the marketplace. The random errors are thought to represent information unobservable to the researcher. This unobservable information could be omitted characteristics that make particular alternatives more attractive than others. In a time series context, the omitted variables could be inventory which affects the marginal utility of consumption. In a conjoint survey exercise, respondents are explicitly asked to make choices solely on the basis of attributes and levels presented and to assume that all other omitted characteristics are the same. It might be argued that there role of random utility errors is different in the conjoint context. Random utility errors might be more the result of measurement error rather than omitted variables that influence the marginal utility of each alternative.

³That is, if price takes on K values, p_1, \dots, p_K , then many conjoint investigators include $K - 1$ dummy variables for each of the values. This makes the market demand a non-continuous function and can create a situation in which there does not exist an equilibrium price. Existence of pure strategy equilibria requires (at a minimum) a continuous best-response function. If utility is a discontinuous function of price, then there can be discontinuities in the best response functions. As our proposed method requires equilibrium calculations, we do not use a dummy variable coding. Nonlinearities in the utility function with respect to price can be handled via non-linear continuous functions, if desired.

However, even in conjoint setting, we believe it is still possible to interpret the random utility errors as representing a source of unobservable utility. For example, conjoint studies often include brand names as attributes. In these situations, respondents may infer that other characteristics correlated with the brand name are present even though the survey instructions tell them not to make these attributions. One can also interpret the random utility errors as arising from functional form misspecification. That is, we know that the assumption of a linear utility model (no curvature and no interactions between attributes) is a simplification at best. We can also take the point of view that a consumer is evaluating a choice set prior to the realization of the random utility errors which occurs during the purchase period. For example, consider the value of choice in the smartphone category at some point prior to a purchase decision. At this point, the consumer knows the distribution of random utility errors which will depend on features not yet discovered or from demand for features which is not yet realized (i.e. the benefit from a better browser is not known with certainty prior to choice). When the consumer actually purchases a smartphone, he/she will know the realization of these random utility errors.

To derive the standard choice model, we must calculate the probability that the j th alternative has the maximum utility, employing the assumption that the error terms have a specific distribution. As is well known, the utility index in Eq. 2.2 is arbitrary and is preserved under both a location and scale shift. That is, if any number is subtracted from the utility of all choice alternatives, then the index of the maximum remains unchanged. Another way of saying this is that utility can only be expressed in a relative sense and that utility is not on a ratio scale. It is also true that the index of the maximum utility is not changed if each alternative is scaled by the same positive number. In conjoint applications with dummy variable coding the attributes (each level except one is introduced via dummy variables), there is one level of all variables (the “default” or base level) which is assigned a utility of zero. This means that, in conjoint designs, there is no location invariance problem. However, there is still a scaling problem. Typically, researchers set the scale parameter of the Type I extreme value distribution to 1.0. Another approach to the scaling invariance problem is to set the price coefficient to the value -1.0 and estimate the scale parameter. This latter approach may have some advantages as Sonnier et al. (2007) point out. However, all of the conjoint studies we are familiar with use the conventional restriction of setting the scale parameter to 1.0 and allowing for a price coefficient. This means that absolute value of the price coefficient should be interpreted as the reciprocal of the error scale parameter.

Assuming that every consumer has sufficient budget to purchase any alternative, the random utility model yields that standard logit specification commonly used to analyze choice-based conjoint with the scale parameter set to 1.0.

$$\Pr(j) = \frac{\exp(\beta' a_j - \beta_p p_j)}{\sum_{j=1}^J \exp(\beta' a_j - \beta_p p_j)} \quad (2.3)$$

To simplify notation, we will combine both β and β_p into one vector, denoted β . In the conjoint literature, the β coefficients are called part-worths. It should be noted that the part-worths are expressed in a utility scale which has an arbitrary origin (as

defined by the base alternative) and an equally arbitrary scaling (somewhat like the temperature scale). This means that we cannot compare elements of the β vector in ratio terms or utilizing percentages. In addition, if different consumers have different utility functions (which is almost a truism of marketing) then we cannot compare part-worths across individuals. For example, suppose that one respondent gets twice as much utility from feature A as feature B, while another respondent gets three times as much utility from feature B as A. All we can say is that the first respondent ranks A over B and the second ranks B over A; no statements can be made regarding the relative “liking” of the various features.

The aggregate demand facing the firm is simply the integral of Eq. 2.3 over the distribution of preferences.

$$q_j(p) = M \int \Pr(j|\beta, A, p) \delta(\beta|\Theta) d\beta$$

Here β refers to the entire vector part-worths (including the price coefficient). $\delta(\bullet)$ is the probability density function of preferences, indexed by parameters, Θ . M is the size of the market. p is the vector of prices for all J products. A is the current choice set of competitive offerings in the market. That is, the j th row of A contains the attributes/characteristics of the j th product in the market.

$$A = \begin{bmatrix} a'_1 \\ a'_2 \\ \vdots \\ a'_J \end{bmatrix}$$

Our equilibrium calculations will depend on the demand for products in a market place in which one of the products is enhanced or augmented with a specific feature. We denote the set of products in the world with feature enhancement by A^* and the set without any feature enhancement as A .

2.3 Computing equilibrium prices

The standard static Nash equilibrium in a market for differentiated products is a set of prices that simultaneously satisfy all firms profit-maximization conditions. Each firm chooses price to maximize firms profits, given the prices of all other firms. These conditional demand curves are sometimes called the “best response” of the firm to the prices of other firms. An equilibrium, if it exists,⁴ is a set of prices that are simultaneously the best response or profit maximizing for each firm given the others.

In a choice setting, the firm profit function⁵ is

$$\pi(p_j|p_{-j}) = M\mathbb{E}[\Pr(j|p, A)](p_j - c_j). \quad (2.4)$$

⁴There is no guarantee that a Nash equilibrium exists for heterogeneous logit demand.

⁵Again, we do not have an aggregate demand shock in the model. We think of the firm problem as setting prices given the observed characteristics and prices of all products in the marketplace. There is no sense in which firms are setting prices as a function of some unobserved characteristic as this is explicitly ruled out by the nature of the conjoint randomized experiment.

M is the size of the market, p is the vector of the prices of all J firms in the market, c_j is the marginal cost of producing the firm's product. The expectation is taken with respect to the distribution of choice model parameters. In the logit case,⁶

$$\mathbb{E}[\Pr(j|p, A)] = \int \frac{\exp(\beta' a_j - \beta_p p_j)}{\sum_j \exp(\beta' a_j - \beta_p p_j)} \delta(\beta, \beta_p) d\beta d\beta_p. \quad (2.5)$$

The first order conditions of the firm are

$$\frac{\partial \pi}{\partial p_j} = \mathbb{E} \left[\frac{\partial}{\partial p_j} \Pr(j|p, A) \right] (p_j - c_j) + \mathbb{E}[\Pr(j|p, A)] \quad (2.6)$$

The Nash equilibrium price vector is a root of the system of nonlinear equations which define the F.O.C. for all J firms. That is if we define

$$h(p) = \begin{bmatrix} h_1(p) = \frac{\partial \pi}{\partial p_1} \\ h_2(p) = \frac{\partial \pi}{\partial p_2} \\ \vdots \\ h_J(p) = \frac{\partial \pi}{\partial p_J} \end{bmatrix} \quad (2.7)$$

then the equilibrium price vector, p^* , is a zero of the function $h(p)$.

There are two computational issues that arise in the calculation of Nash equilibrium prices. First, both the firm profit function (2.4) and the FOC conditions for the firm (2.6) require the computation of integrals to compute the expectation of the market share (market demand) and expectation of the derivative of market share in the FOC. Second, an algorithm must be devised for calculating the equilibrium price, given a method of approximating the integrals. The most straightforward method to approximate the requisite integrals is a simulation method. Given a distribution of demand parameters over consumers, we can approximate the expectations by a simple average of draws from this distribution. Given that both the market share and the derivatives of market share are virtually costless to evaluate, an extremely large number of draws can be used to approximate the integrals (we routinely use in excess of 50,000 draws).

Given the method for approximating the integral, we must choose an iterative method for computing equilibrium prices. There are two methods available. The first is an iterative method where we start from some price vector, compute the optimal price for each firm given other prices, updating the price vector as we progress from the 1st to the J th firm. After one cycle through the J firms, we have updated the price vector to a second guess of the equilibrium. We continue this process until $\|p^r - p^{r-1}\| < tol$. The method of iterative firm profit maximization will work if

⁶We do not include a market wide shock to demand as we are not trying to build an empirical model of market shares. We are trying to approximate the firm problem. In a conjoint setting, we abstract from the problem of omitted characteristics as the products we use in our market simulators are defined only in terms of known and observable characteristics. Thus, the standard interpretation of the market wide shock is not applicable here. Another interpretation is that the market wide shock represents some sort of marketing action by the firms (e.g. advertising). Here we are directly solving the firm pricing problem holding fixed any other marketing actions. This means that the second interpretation of the market wide shock as stemming from some unobservable firm action is not applicable here.

there is a stable equilibrium. That is, if we perturb the price vector away from the equilibrium price, the iterative process will return to the equilibrium (at least in a neighborhood of the equilibrium). This is not guaranteed to occur even if there exists a unique equilibrium.

The second method for computing equilibrium prices is to find the root of the set of FOCs (2.7). The optimization problem

$$\min_p \|h(p)\|$$

can be solved via a quasi-Newton method which is equivalent to finding the roots directly using Newton's method with line search. This provides an alternative way of finding equilibria, if they exist, but does not provide a way of finding the complete set of equilibria if multiple equilibria exist. The zeroes of the set of first order conditions are not guaranteed to be equilibria and we must check candidate roots to see if they are indeed profit maximizing for the firm. In practice, we use both the iterative best response method and the root finding method to check all of our solutions to insure that they represent valid equilibria. The existence of multiple equilibria would have to be demonstrated by construction via starting the optimizer/root finder from different starting points. In our experience with heterogeneous logit models, we have not found any instance of multiple equilibria; however we have found situations where we cannot find any equilibria (though only rarely and for extreme parameter values).

2.4 WTP measures of feature value

Willingness to Pay (WTP) is a demand-based measure of social welfare used to value product features. We introduce it here for comparison to an equilibrium-based profits approach. WTP is a measure of social welfare derived from the principle of compensating variation. That is, WTP for a product is the amount of income that will compensate for the loss of utility obtained from the product; in other words, a consumer should be indifferent between having the product or not having the product with an additional income equal to the WTP. Indifference means the same level of utility. For choice sets, we must consider the amount of income (called the compensating variation) that must be paid to a consumer faced with a diminished choice set (either an alternative is missing or diminished by omission of a feature) so that consumer attains the same level of utility as a consumer facing a better choice set (with the alternative restored or with the feature added). Consumers evaluate choices *a priori* or before choices are made. Features are valuable to the extent to which they enhance the attainable utility of choice. Consumers do not know the realization of the random utility errors *until* confronted with the specific choice tasks and the description of the choice alternatives. Addition of the feature shifts the deterministic portion of utility or the mean of the random utility. Variation around the mean due to the random utility errors is equally important as a source of value.

To evaluate the utility afforded by a choice set, we must consider the distribution of the maximum utility obtained across all choice alternatives. This maximum has a distribution because of the random utility errors. For example, suppose we add the feature to a product configuration that is far from utility maximizing. It may

still be that, even with the feature, the maximum deterministic utility is provided by a choice alternative without the feature. This does not mean that feature has no value simply because the product it is being added to is dominated by other alternatives in terms of deterministic utility. The alternative with the featured added can be chosen after realization of the random utility errors if the realization of the random utility error is very high for the alternative that is enhanced by addition of the feature.

More formally, we can define WTP from a feature enhancement by using the indirect utility function associated with the choice problem. Let A be a matrix which defines the set of products in a choice set. A is a $J \times K$ matrix, where J is the number of choice alternatives and K is the number of attributes which define each choice alternative (other than price). The rows of the choice set matrix, a_j , show the configuration of attributes for the j th product in the choice set. That is, the j th row of A defines a particular product – a combination of attribute levels for each of K attributes. If the k th attribute is the feature in question, then $a_{j,k} = 1$ implies that the feature has been added to the j th product. Let A denote a set of products which represent the marketplace without the new feature and A^* denotes the same set of products but where one of the products has been enhanced by adding the feature. We define the indirect utility function for a given choice set as

$$V(p, y|A) = \max_x U(x|A) \quad \text{subject to } p'x \leq y \quad (2.8)$$

WTP is defined as the compensating variation required to make the utility derived from the feature-poor choice set, A , equal to the utility obtained from the feature-rich choice set, A^* . $U(\cdot)$ is a standard direct utility function defined over the vector, x , of consumption of goods.

$$V(p, y + WTP|A) = V(p, y|A^*) \quad (2.9)$$

As such, WTP is a measure of the social welfare conferred by the feature enhancement expressed in dollar terms. The choice set may include not only products defined by the K product attributes but also an outside option which is coded as row of zeroes in the feature matrix and a price of 0. Thus, a consumer receives utility from three sources: 1. observed characteristics of the set of products in the market, 2. expenditure on a possible outside alternative and 3. the random utility error.

For the logit demand system, the indirect utility function is obtained by finding the expectation of the maximum utility (see, for example, McFadden (1981)).

$$\begin{aligned} V(p, y|A) &= E[\max_j U_j|A] \\ &= \beta_p y + \ln \sum_{j=1}^J \exp(a'_j \beta - \beta_p p_j) \end{aligned} \quad (2.10)$$

To translate this utility value into monetary terms, we divide by the marginal utility of income. In these models, the price coefficient is viewed as the marginal utility of

income. We can then transform the utility value in Eq. 2.10 in to monetary terms, resulting in the “social surplus” (Trajtenberg 1989).

$$W(A|p, \beta, \beta_p) = y + \ln \left[\sum_{j=1}^J \exp(\beta' a_j - \beta_p p_j) \right] / \beta_p \quad (2.11)$$

We can then solve for WTP using the Eq. 2.9.

$$WTP = \ln \left[\sum_{j=1}^J \exp(\beta' a_j^* - \beta_p p_j) \right] / \beta_p - \ln \left[\sum_{j=1}^J \exp(\beta' a_j - \beta_p p_j) \right] / \beta_p \quad (2.12)$$

In the conjoint literature (Orme 2001), another “WTP” measure is often used. If we simply scale the part-worth corresponding to the feature enhancement by the price coefficient, a monetary measure of the incremental utility afforded by the feature enhancement is obtained. This measure does not take into account the utility value afforded by the omitted characteristics represented by the random utility errors. In addition, this measure, which we term a “pseudo-WTP” measure is invariant to which product the feature enhancement is applied to a property not shared by either a true WTP measure or the incremental profits measure proposed here. Thus, while the “pseudo-WTP” measure might be useful as a way of interpreting and comparing coefficients in conjoint models, there is no rigorous basis for the use of this measure as a measure of social surplus.

3 Using conjoint designs for equilibrium calculations

Economic valuation of feature enhancement requires a valid and realistic demand system as well as cost information and assumptions about the set of competitive products. In order to use a conjoint survey as the basis for calibration of a valid demand system, we must use the choice-based conjoint method. That is, we must measure demand (product choice) at the respondent level. In some forms of conjoint data, respondents are asked to rate various product configurations or to allocation 100 points across the configurations according to their “preferences.” These methods have fallen out of favor relative to choice based conjoint as they don’t provide a realistic simulation of the marketplace in which consumers choose among different products.

If conjoint studies are to be used to calibrate the demand system, then particular care must be taken to design a realistic conjoint exercise. The low cost of fielding and analyzing a conjoint design makes this method particularly appealing. However, there is no substitute for careful conjoint design. Many designs fielded today are not useful for economic valuation of feature enhancement. For example, conjoint studies in which there is no outside option, only one brand, and only a small subset of product features are often used in commercial applications. A study with any of these limitations is of questionable value for true economic valuation.

Practitioners of conjoint have long been aware that conjoint is appealing because of its simplicity and low cost but that careful studies make all the difference between realistic predictions of demand and useless results. We will not repeat the many prescriptions for careful survey analysis which include crafting questionnaires with terminology that is meaningful to respondents, thorough and documented pre-testing, and representative (projectable) samples.⁷ Furthermore, many of the prescriptions for conjoint design including well-specified and meaningful attributes and levels are extremely important. Instead, we will focus on the areas we feel are especially important for economic valuation and not considered carefully enough.

It is also well known that conjoint studies can be used to assess the distribution of preferences for products and features but are silent regarding awareness and availability. That is, the conjoint exercise makes the consumers (survey respondents) aware of the new product features and assumes that all choice alternatives are, hypothetically at least, available for purchase. For some consumer products, issues of awareness and availability (distribution) are extremely important. For consumer electronics examples, such as the one featured here, availability is typically not a problem, but awareness may be still be, particularly for minor feature enhancements. Our results for feature valuation should be viewed, therefore, as the maximum possible value under the assumption that, in the long run, consumers would become aware of a product feature which has utility value.

3.1 Set of competing products

The guiding principle in conjoint design for economic valuation of feature enhancement is that the conjoint survey must closely approximate the marketplace confronting consumers. In industry applications, the feature enhancement has typically not yet been introduced into the marketplace (hence the appeal of a conjoint study).

Most practitioners of conjoint are aware that, for realistic market simulations, the major competing products must be used. This means that the product attributes in the study should include not only functional attributes such as screen size, memory etc but also the major brands. This point is articulated well in Orme (2001).

In a highly competitive product category with many highly substitutable products, the economic value or increment profits that could accrue to any one competitor would typically be very small. However, in an isolated part of the product space (that is a part of the attribute space that is not densely filled in with competing products), a firm may capture more of the value to consumers of a feature enhancement. Therefore, it is important to design the study to consider the major competing products both in terms of brands and the attributes used in the conjoint design. It is not required that the conjoint study exactly mirror the complete set of products and brands that are in the marketplace but that the main exemplars of competing brands and product positions must be included.

⁷For a general discussion of surveys and sampling, see Rossi et al. (1983). For specific guidelines on the development of conjoint surveys see Orme (2009).

3.2 Outside option

There is considerable debate as to the merits of including an outside option in conjoint studies. Many practitioners use a “forced-choice” conjoint design in which respondents are forced to choose one from the set product profiles in each conjoint choice task. The view is that “forced-choice” will elicit more information from the respondents about the tradeoffs between product attributes. If the “outside” or “none of the above” option is included, advocates of forced choice argue that respondents may shy away from the cognitively more demanding task of assessing tradeoffs and select the “none” option to reduce cognitive effort. On the opposite side of the argument, some practitioners advocate inclusion of the outside option in order to assess whether or not the product profiles used in the conjoint study are realistic in the sense of attracting considerable demand. The idea being that if respondents select the “none of the above” option too frequently then the conjoint design has offered very unattractive hypothetical products. Others (see, for example, Brazell et al. (2006)) argue the opposite side of the argument for forced choice. They argue that there is a “demand” effect in which respondents select at least one product to “please” the investigator. There is also a large literature on how to implement the outside option.

Whether or not the outside option is included depends on the ultimate use of the conjoint study. Clearly, it is possible to measure how respondents trade-off different product attributes against each other without inclusion of the outside option. For example, it is possible to estimate the price coefficient in a conjoint study which does not include the outside option. Under the assumption that all respondents are NOT budget constrained, the price coefficient should theoretically measure the trade-offs between other attributes and price. The fact that respondents might select a lower price and pass on some features means that they have an implicit valuation of the dollar savings involved in this trade-off. If all respondents are standard economic agents, then this valuation of dollar savings is a valid estimate of the marginal utility of income.

While demand parameters can, in principle, be measured from a conjoint study conducted without the outside option, valid equilibrium calculations do require an outside alternative. In order to compute valid equilibrium prices, we need to explicitly consider substitution from and to other goods including the outside good. For example, suppose we enhance a product with a very valuable new feature. We would expect to capture sales from other products in the category as well as to expand the category sales; the introduction of the Apple iPad dramatically grew the tablet category due, in part, to the features incorporated in the iPad. Chintagunta and Nair (2011) make a related observation that price elasticities will be biased if the outside option is not included.

We conclude that an outside option is essential for economic valuation of feature enhancement as the only way to incorporate substitution in and out of the category is by the addition of the outside option. At this point, it is possible to take the view that if respondents are pure economic actors that they should select the outside option corresponding to their true preferences and that their choices will properly reflect the marginal utility of income. However, there is a growing literature which suggests that different ways of expressing or allowing for the outside option will change the

frequency with which it is selected. In particular, the so-called “dual response” way of allowing for the outside option (see Uldry et al. (2002) and Brazell et al. (2006)) has been found to increase the frequency of selection of the outside option. The “dual-response” method asks the respondent first to indicate which of the product profiles (without the outside option) are most preferred and then asked if the respondent would actually buy the product at the price posted in the conjoint design. Our own experience confirms that this mode of including the outside option greatly increases the selection of the outside option. Our experience has also been that the traditional method of including the outside option often elicits a very low rate of selection which we view as unrealistic. The advocates of the “dual response” method argue that the method helps to reduce a conjoint survey bias toward higher purchase rates than in the actual marketplace.

Another way of reducing bias toward higher purchase rates is to design a conjoint using an “incentive-compatible” scheme in which the conjoint responses have real monetary consequences. There are a number of ways to do this (see, for example, Ding et al. (2005)) but most suggestions (an interesting exception is Dong et al. (2010)) use some sort of actual product and a monetary allotment. If the products in the study are actual products in the marketplace, then the respondent might actually receive the product chosen (or, perhaps, be eligible for a lottery which would award the product with some probability). If the respondent selects the outside option, they would receive a cash transfer (or equivalent lottery eligibility). Incentive compatible conjoint methods are difficult to implement in the case of new products or new product features for which no actual product may be available.

4 Statistical inference for economic valuation

Bayesian hierarchical models are now by far the dominant method for use in analysis of choice-based conjoint data. The reason for the widespread use of Bayesian methods is the ability to conduct inference at both the individual consumer or respondent level as well as on the aggregate level. All inferences are made without resort to asymptotic approximations which can be very poor for highly nonlinear models and functions of model parameters. For example, the posterior distribution of equilibrium profits involves computing Nash Equilibrium prices. Equilibrium prices are a highly nonlinear function of the distribution of preferences in the marketplace, an expression which is not available in closed form. As is well known, a simulation-based Bayesian approach can compute the finite sample distribution of any function (in closed form or not) of model parameters. In our case, we must compute the posterior predictive distribution of market share and use this distribution to simulate the posterior distribution of equilibrium prices and profits.

We employ a standard hierarchical Multinomial Logit model. This consists of a MNL model for each respondent's data (denoted y_i) given attribute information, A_i , and preference parameters, β_i , coupled with a two-stage prior. The first stage of the prior is a multivariate normal distribution for the model parameters and the second stage is a set of priors on the parameters of the random coefficient distribution parameters (in Section 6.3, we consider a mixture of normals specification for the first

stage prior or random coefficient distribution). In non-Bayesian literature it is common to restrict heterogeneity to only a subset of the model parameters or to assume that the model parameters do not have a full co-variance matrix, something that is unnecessary in a full Bayesian treatment. Our hierarchical model is specified as

$$\begin{aligned} y_i &| A_i, \beta_i \\ \beta_i &\sim N(\mu, V_\beta) \\ \mu, V_\beta &\sim p(h). \end{aligned} \quad (4.1)$$

The second-stage prior is the standard Normal-Inverted-Wishart conditionally conjugate prior (see, for example, Rossi et al. (2005), Section 2.12).

4.1 Individual or market quantities?

The appeal of Bayesian methods is based primarily on the ability to produce inferences at the respondent level as well as to free statistical inference from asymptotic approximations which are dubious at best in conjoint exercises where very few (invariably less than 20) observations are collected per respondent. However, there has been a great deal of confusion as to how to develop and use respondent-level inferences. Many simply compute Bayesian estimates at the respondent level based on averages of the MCMC draws of the parameter vector at the respondent level.

$$\hat{\beta}_i = \frac{1}{R} \sum_r \beta_i^r \quad (4.2)$$

Here i is the index of the respondent and r is the index of the MCMC draws for that respondent. While there is nothing inherently wrong with this estimate, the distribution of these estimates across respondents is not a valid estimate of true distribution of preference or part-worth parameters than constitutes the market. Intuitively, we all know that there is a great deal of uncertainty in the respondent level parameter estimates as they are based only on a handful of observations and the Bayes procedures do not borrow a great deal of strength from other observations unless the distribution of heterogeneity is inferred to be very tight. These problems are magnified for the WTP measure. One cannot simply plug in estimates of respondent-level parameters into the equation defining WTP (2.9) and then compute the average of these estimated WTP as an estimate of the population average WTP. A coherent full Bayesian approach requires the definition of the population object of interest (here is is the population average of the WTP measure) and then compute the full posterior distribution of this measure.

Instead of focusing on individual estimates and exposing the attendant problems with these estimates, economic valuation forces the investigator to estimate the distribution of tastes which is then used to compute market demand. To review, if we know the distribution of part-worths (preferences) over respondents we can compute market demand for any firm's product as

$$\text{MS}(j|p, A) = \int \Pr(j|\beta, p, A) \delta(\beta|\Theta) d\beta. \quad (4.3)$$

Here $MS(j)$ is the market share⁸ of product j . However, we do not know the exact distribution of preferences. What we have is a model for preferences (the first stage of the hierarchical model) and inferences about the parameters of this model. We assume that the model parameters (or some transform of them) takes a specific parametric form. In particular, it is common to assume that preferences are normally distributed.

$$\delta(\beta|\Theta) = \phi(\beta|\mu, V_\beta) \quad (4.4)$$

Here $\phi(\bullet)$ is the multivariate normal density and Θ represents the normal density parameters. This means that market share defined in Eq. 4.3 is a function of the hyper-parameters that govern the normal, first-stage or random coefficient, distribution. Some might argue (see, for example, Rossi (2014)) that the normal distribution is overly restrictive as the first-stage of the prior or the random coefficient distribution. In Section 6.3, we explore the sensitivity of our results to the assumption of normality by considering a much more flexible mixture of normals approach.

$$MS(j|\mu, V_\beta) = \int \Pr(j|\beta) \phi(\beta|\mu, V_\beta) d\beta \quad (4.5)$$

The posterior predictive distribution of market share is obtained from the posterior distribution of the normal distribution hyper-parameters, $p(\mu, V_\beta|data)$ used in conjunction with Eq. 4.5. We simply draw from the multivariate normal density given each draw of the hyper-parameters to obtain a draw from the relevant posterior predictive distribution.

$$MS(j)^r = \int \Pr(j|\beta) \phi(\beta|\mu^r, V_\beta^r) d\beta \quad (4.6)$$

This idea can be used to compute the posterior distribution of any quantity including the posterior distribution of equilibrium prices as equilibrium prices are also a function of the random coefficient parameters. In the illustration of our method, we will take draws of the normal hyper-parameters and then for each draw we will solve for the equilibrium prices. This will build up the correct posterior distribution of equilibrium quantities.

4.2 Estimating price sensitivity

Equilibrium prices are sensitive to inferences regarding the price coefficient. If the distribution of price sensitivities puts any mass at all on positive values, then there does not exist a finite equilibrium price. All firms will raise prices infinitely, effectively firing all consumers with negative price sensitivity and make infinite profits on the segment with positive price sensitivity. Most investigators regard positive price coefficients as inconsistent with rational behavior. However, it will be very difficult for a normal model to drive the mass over the positive half line for price sensitivity to a negligible quantity if there is mass near zero on the negative side. We must distinguish uncertainty in posterior inference from irrational behavior. If a number of

⁸In the conjoint literature, this is often termed the “choice share.”

respondents have posteriors for price coefficients that put most mass on positive values, this suggests a design error in the conjoint study; perhaps, respondents are using price as a proxy for the quality of omitted features and ignoring the “all other things equals” survey instructions. In this case, the conjoint data should be discarded and the study re-designed. On the other hand, we may find considerable mass on positive values simply because of the normal assumption and the fact that we have very little information about each respondent. In these situations, we have found it helpful to change the prior or random effect distribution to impose a sign constraint on the price coefficient.

$$\begin{bmatrix} \beta \\ \beta_p = \ln(-\beta_p^*) \end{bmatrix} \sim N(\mu, V_\beta) \quad (4.7)$$

Given that a RW-Metropolis step is used to draw logit parameters, this reparameterization can be implemented trivially in the evaluation of the likelihood function only.⁹ The only change that should be made is in the assessment of the IW prior on V_β . In the default settings, we use a relatively diffuse prior, $V_\beta \sim IW(\nu, \nu V)$, with $V = I$ and we typically use a value of ν which yields a barely proper IW distribution (such as $\nu = \dim(\beta) + 5$). We must recognize that the price element of β is now on a log-scale and it would be prudent to lower the prior diffusion to a more modest level such as .5 rather than leaving that diagonal element at 1.0. In addition, very low values of ν will result in very thick tails and admit extremely small price coefficients. For these reasons, we believe somewhat larger values of ν are appropriate. We use $\nu = \dim(\beta) + 15$. For these conjugate priors, ν can be interpreted as the size of a sample which provided the basis of the prior. Since we have over 300 respondents, the ν values we employ are still only mildly informative in the sense that the actual sample size is about 10 times the prior degrees of freedom.

Figure 1 shows the implied prior distribution on the price coefficient at both “default” prior settings and a tighter prior that uses $\nu = \dim(\beta) + 15$ and the appropriate diagonal value of V set to .5. The top row of histograms shows the standard default prior distribution. The prior distribution of the log-scale parameter, β_p^* is shown on the left and the implied prior distribution for the price coefficient on the right. The low degrees of freedom for the IW distribution and the relatively large value of V imply a left tail of negative values which are large. This mass gets transformed into extremely small price coefficients, barely less than zero. Clearly, this is not a diffuse prior at all but, rather, a very informative prior that puts high prior probability near zero. Our suggested prior is shown in the bottom row of histograms. The prior on β_p^* is has much thinner tails and, therefore, the implied prior on the price coefficient is much more reasonable and puts less mass on extremely values of price sensitivity.

Even with a prior that only puts positive mass on negative values, there may still be difficulties in computing sensible equilibrium prices due to a mass of consumers with negative but very small price sensitivities. Effectively this will flatten out the profit

⁹It should be noted that the prior outlined here will have a zero probability of a price coefficient which is ≥ 0 . This is not true for more ad hoc methods such as the method of “tie-breaking” used in Sawtooth Software.

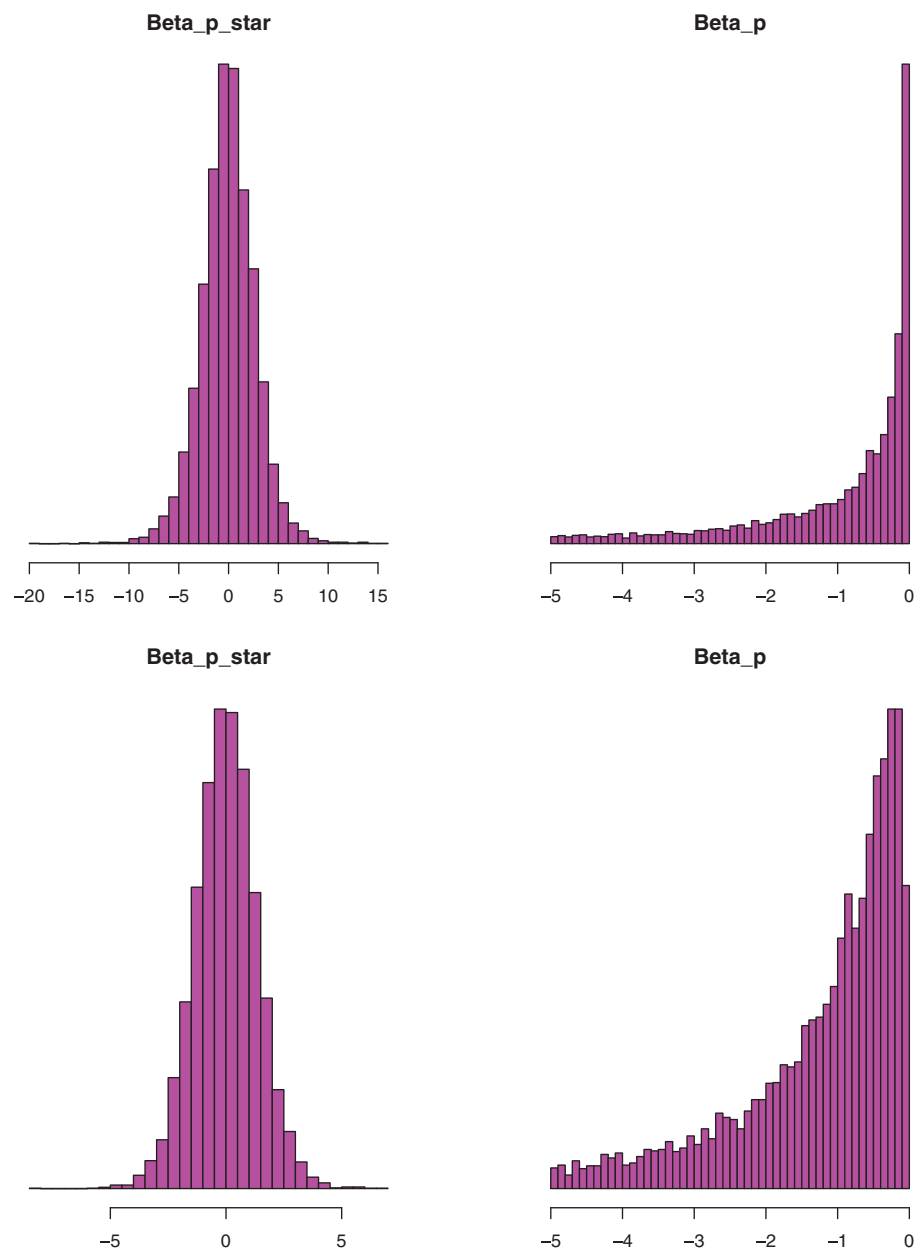


Fig. 1 Default and tighter prior distributions of the price coefficient

function for each firm and make it difficult to find an equilibrium solution. The lower the curvature of the profit function, the more sensitive the profit function (or first order conditions) will be to simulation error in the approximation of the integrals.

In many conjoint studies, the goal is to simulate market shares for some set of products. Market shares can be relatively insensitive to the distribution of the price coefficients when prices are fixed to values typically encountered in the marketplace. It is only when one considers relative prices that are unusual or relatively high or low prices that the implications of a distribution of price sensitivity will be felt. By definition, price optimization will stress-test the conjoint exercise by considering prices outside the small range usually consider in market simulators. For this reason, the quality standards for design and analysis of conjoint data have to be much higher when used for economic valuation than for many of the typical uses for conjoint. Unless the distribution of price sensitivity puts little mass near zero, the conjoint data will not be useful for economic valuation using either our equilibrium approach or for the use of the more traditional pseudo-WTP methods.

5 Decision-theoretic considerations and “optimal” pricing

Our approach has been to compute equilibrium prices under the assumption that firms are fully informed regarding not only the form of the distribution of preferences but also the parameters of this distribution. We, as the researcher, do not have full information and must make inferences regarding what equilibrium prices would be under the assumption of fully informed firms and on the basis of the sample (conjoint data) information. If we denote the generic distribution of consumer preferences as, $\delta(\beta|\Theta)$, then the set of equilibrium prices that simultaneously satisfy all J firms’ first order conditions (2.7) are function of Θ , denoted $p^*(\Theta)$. Using draws from the posterior distribution of Θ , we can simulate the posterior distribution of p^* or any other function of equilibrium prices as discussed in Section 4. Our feature valuation metric is the incremental profits that accrue to products with enhanced features and we can compute the posterior distribution of incremental profits given our assumption that firms have full information and that there is a Nash equilibrium in a pricing game.

The posterior mean of equilibrium prices (the standard Bayes estimator) is not the solution to the “optimal” pricing problem of the firm under the assumption that the firm’s information regarding preferences is limited to conjoint sample information. If there was only one firm, the firm would solve a decision-theoretic problem of maximizing expected profits where the expectation is taken with respect to the posterior distribution of preference parameters.

$$\max_p \bar{\pi}(p) = \int \pi(p|\Theta) p(\Theta|Data) d\Theta \quad (5.1)$$

Here $\pi(p|\Theta) = \int Pr(j|\beta) (p - c) \delta(\beta|\Theta) d\beta$.

The solution to this problem in Eq. 5.1 is not the same finding the optimal price given Θ and then taking the average of these optimal prices with respect to the posterior distribution of Θ . The problem defined above provides the solution to the optimal pricing problem of the monopolist and results in only one value of an “optimal” price.

However, if there is more than one firm in the industry (as is invariably the case with differentiated products), we cannot simply apply the decision theoretic paradigm and provide advice regarding optimal price to the firm without further assumptions. One simple solution would be to make some assumptions regarding the prices of other products and simply condition on them in the solution to the decision theoretic problem. This, of course, does not recognize that as features are added to products there is likely to be a competitive price response and that we must make assumptions regarding the prices of competing products both in the world with and without the product feature enhancement. Rather than making arbitrary assumptions, our contribution is to use equilibrium notions to solve this problem.

In order to apply equilibrium concepts to the situation where the firm does not have full information regarding the distribution of preferences, we must make assumptions regarding what information is available to other firms and define an equilibrium for the pricing game with these information sets. Progress along these lines could be made in two ways: 1. we could assume that all firms have the same information set, namely our conjoint data and 2. that the focal firm is the only firm with access to the conjoint data and all other firms have information equivalent to that which provides the prior for the focal firm. If we assume that all firms have the same posterior distribution over preference parameters given by the posterior from our conjoint data, then we can define a Nash equilibrium by finding the zero of the set of first order conditions based on integrating out Θ with respect to the posterior derived from this common information set. While the symmetry of the assumption of the same information set is mathematically convenient, it may be hard to justify this assumption. One would have to argue that all firms are aware of the possibility of the feature enhancement in question and have the resources to conduct similar conjoint surveys. The second situation where there are different information sets for different firms presents theoretical and computational challenges which we leave for future research.

Some would argue that the assumption that all firms have full information about the distribution of consumer preferences is particularly unrealistic in the case of new product features. In the case of new product features, there is often little or no marketplace data. In fact, the widespread commercial use of conjoint surveys for feature valuation indicates that many firms do not fully know the distribution of consumer preferences. It is important to understand that our valuation calculations are not prescriptive. We are not claiming to solve the optimal pricing problem of the firm, given limited information. Our view is that the valuation computations based on profits in equilibrium with full information provides an approximation to a somewhat longer run view of firm profits. That is, we would expect that firms will introduce products into the marketplace with the features in question and that, overtime, all firms in the marketplace will learn from both survey and marketplace data regarding the distribution of consumer preferences. In the longer run, the sequence of industry equilibria could be expected to converge to the full information Nash equilibrium calculated here. In that sense, the firm today, with only the results of the conjoint survey, is like the researcher studying industry equilibrium with fully informed firm.

The firm can form posterior beliefs about the eventual industry outcome which can be approximated by the full information Nash equilibrium.

6 An illustration using the digital camera market

To illustrate our proposed method for economic valuation and to contrast our method with standard WTP methods, we consider the example of the digital camera market. We designed a conjoint survey to estimate the demand for features in the point and shoot submarket. We considered the following seven features with associated levels:

1. Brand: Canon, Sony, Nikon, Panasonic
2. Pixels: 10, 16 mega-pixels
3. Zoom: 4×, 10× optical
4. Video: HD (720p), Full HD (1080p) and mike
5. Swivel Screen: No, Yes
6. WiFi: No, Yes
7. Price: \$79–279

We focused on evaluating the economic value of the swivel screen feature which is illustrated in Fig. 2. The conjoint design was a standard fractional factorial design in which each respondent viewed sixteen choice sets, each of which featured four hypothetical products. A dual response mode was used to incorporate the outside option. Respondents were first asked which of the four profiles presented in each choice task was most preferred. Then the respondent was asked if they would buy the preferred profile at the stated price. If no, then this response is recorded as the “outside option” or “none of the above.” Respondents were screened to only those who owned a point and shoot digital camera and who considered themselves to be a major contributor to the decision to purchase this camera.

The survey was fielded to the Sampling Surveys International internet panel in August 2013. We received 501 completed questionnaires.¹⁰ We recorded time to complete the conjoint portion of the survey. The median time to complete is 220 seconds or about 14 seconds per conjoint task. The 25th percentile is 151 seconds and the 75th percentile is 333 seconds. To check sensitivity to time spent on the survey, we conducted analyses deleting the bottom quartile of the respondents and found little change. It is a common and well-accepted practice to remove respondents who “straight-line” or always select the same option (such as the left most choice). The idea is that these “straight-liners” are not putting sufficient effort into the choice task. Of our 501 complete questionnaires, only two respondents displayed straight-line behavior and were eliminated. We also eliminated six respondents who always selected the same brand and one respondent who always selected the high

¹⁰This study was part of a wave of four other very similar conjoint studies on digital cameras each with the same screening criteria. For all studies in the wave, 16,185 invitations were sent to panelists, 6,384 responded. Of those who responded to the invitation, 2,818 passed screening and of those passing screening 2,503 completed the questionnaire. Thus, the overall completion rate is 89 per cent which is good by survey standards.



Fig. 2 Swivel screen attribute

price brand. Our reasoning is that these respondents did not appear to be taking the trade-offs conjoint exercise seriously. We also eliminated 23 respondents who always selected the outside option as their part-worths are not identified without prior information.

The remaining 469 respondents were considered for inclusion in our analysis. Conjoint survey experts frequently impose further screening criteria in order to eliminate respondents who appear to be doing little more than providing random responses to the survey. Typically this is done by fitting the logit model to the complete data set and then removing respondents whose hit rate or log-likelihood values are barely above chance (in our case that would be a 1/5 probability of selecting one of the four brands and the outside option). We are reluctant to impose such criteria for selection into our estimation sample. However, we recognize that this means that our sample of respondents will contain some respondents who appear to be rather price insensitive. This may lower overall own price elasticities and result in somewhat higher margins. In Section 6.3, we will explore the sensitivity of our results to both the criteria used to select respondents as well as the form of the first-stage prior for heterogeneity.

To analyze the conjoint data, we use the *bayesm* routine `rhierMnlMixture`. We employed standard diffuse prior settings as discussed in Section 4 and 50,000 MCMC draws were made. The first 10,000 draws were discarded for burn-in purposes. The hierarchical model we employed assumes that the conjoint price-worths are normally distributed. We can compute the posterior predictive distribution of part-worths as follows:

$$p(\beta|data) = \int \phi(\beta|\mu, V_\beta) p(\mu, V_\beta|data) d\mu dV_\beta \quad (6.1)$$

Equation 6.1 shows the influence of both the model (the normal random coefficient distribution) and the data through the posterior distribution of the normal hyperparameters. The resulting distributions will be symmetric but of fatter tails than the normal. Figure 3 shows the posterior predictive distribution of the swivel screen part-worth. Most the mass of this distribution is on positive values. It is difficult to interpret the size of these part-worths without reference to the price coefficient. Figure 3 displays the posterior predictive distribution of the price coefficient. This coefficient has been restricted to only negative values by the reparameterization in Eq. 4.7. Note that this is the price coefficient for price expressed in \$100s.

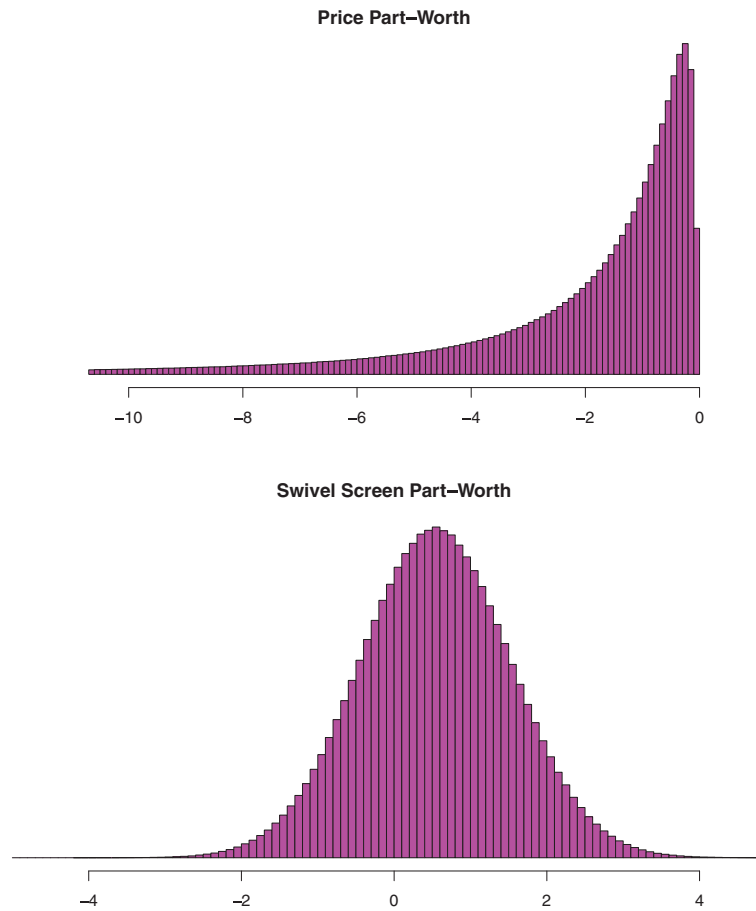


Fig. 3 Posterior predictive distribution of price and swivel screen part-worths

Aggregate demand is found by taking the expectation of choice probabilities with respect to the distribution of preference parameters over the population. The distribution of the part-worths shown in Fig. 3 is the correct predictive distribution of preferences. In order to shed some light as to the substitution structures found in aggregate demand, we compute the posterior distribution of the market share elasticity matrix. This is obtained by computing the elasticity matrix given the normal distribution hyper-parameters and then constructing the posterior distribution of this quantity using draws from the posterior distribution of the hyper-parameters.

$$\frac{\partial MS(i)}{\partial \ln p_j}(\mu, V_{beta}) = \frac{\partial}{\partial \ln p_j} \int Pr(i|\beta) \phi(\beta|\mu, V_{beta}) d\beta \quad (6.2)$$

The posterior mean of these elasticities is presented in Table 1. These own price elasticities, while plausible, imply a reasonably high markup of two- three times cost.

Table 1 Posterior mean of aggregate demand elasticities

Price Mkt Share	MS_{Sony}	MS_{Canon}	MS_{Nikon}	$MS_{Panasonic}$
P_{Sony}	-1.78	.51	.39	.45
P_{Canon}	.48	-1.69	.36	.35
P_{Nikon}	.40	.40	-1.61	.30
$P_{Panasonic}$.55	.45	.36	-1.75

The cross-price elasticities are also quite high, showing a high degree of substitution between these brands.

6.1 Changes in equilibrium prices, shares, and profits

We have argued that economic value should be expressed as incremental profits that accrue to the firm that engages in feature enhancement. It is difficult to provide a realistic base or scaling for firm profits without more information regarding market size and cost. However, we can compute equilibrium prices with and without feature enhancement to provide an idea of how much the focal firm can charge as a price premium and how market shares will adjust in the new industry equilibrium. Here we consider the change in equilibrium outcomes from adding the swivel screen display to the Sony base product (a Sony brand camera with all attributes turned to their “lowest” value except, of course, price which is not constrained). The value conferred by the addition of the swivel screen will also depend on the configuration of other competing products. For illustration purposes only, we considered a competitive set that consists of three other brands (Canon, Nikon, and Panasonic) all similarly configured at the “base” level of attributes. We set the marginal cost of product for all brands to be \$75. When the Swivel Screen feature is added, we assume marginal cost is increased by \$5 to \$80.

Table 2 presents the posterior means of the equilibrium prices computed with and without the swivel screen addition to the Sony product.¹¹ As we might expect, adding the swivel screen gives the Sony brand more effective market power relative to the other branded competitors who do not have the feature (note: we could have easily simulated a competitive reaction in which some or all of the other brands adopted the feature). Not only does Sony find it optimal to raise price, the stronger competition and diminished value of the other brands forces them to lower prices in equilibrium.

Figure 4 plots the posterior distribution of the change in equilibrium price as the swivel screen feature is added to the Sony product. This distribution is distributed around the mean of \$34.42, represented by the single light-shaded vertical bar. The two dark-shaded vertical bars on either side of the mean represent a 95 per cent posterior interval.

¹¹The numbers displayed in the table are posterior means.

Table 2 Changes in equilibrium prices

	Sony	Canon	Nikon	Panasonic
W/O SS	\$172.06	\$186.52	\$203.10	\$178.30
W SS	\$206.49	\$181.53	\$192.47	\$173.34
Δ	\$34.42	−\$4.98	−\$10.63	−\$4.95

Table 3 displays the equilibrium market shares for each of the four branded products and the outside good calculated with and without the swivel screen display. Only very minor share changes are observed. The feature enhanced Sony product

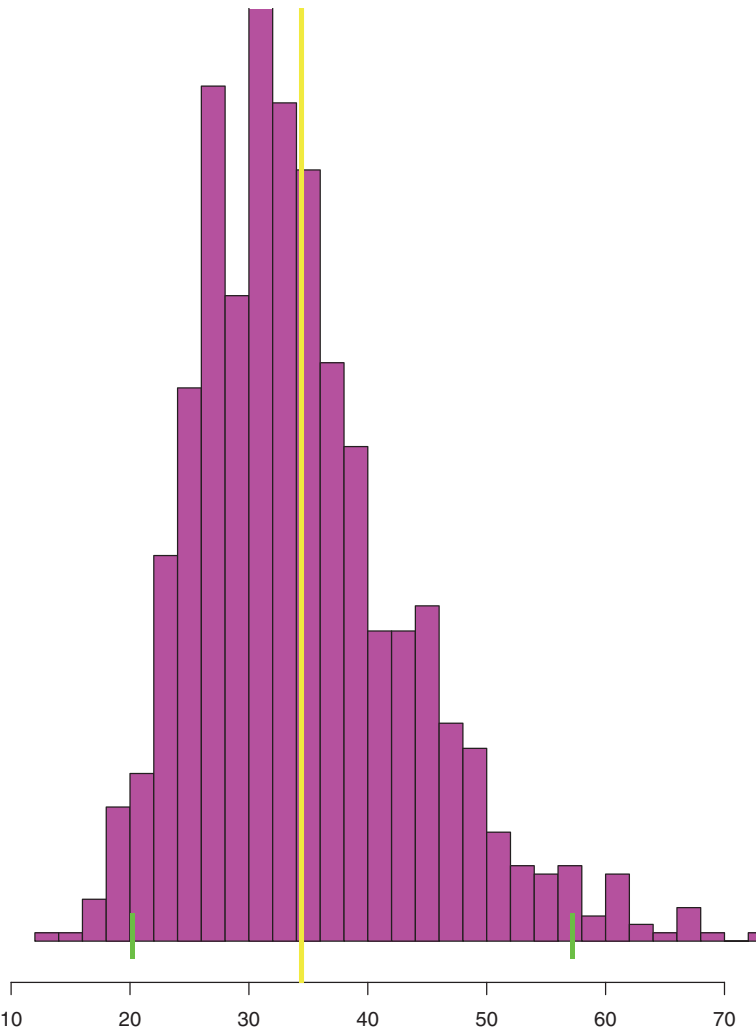


Fig. 4 Posterior distribution of the change in Sony equilibrium price

Table 3 Changes in equilibrium shares

	Sony	Canon	Nikon	Panasonic	Outside Good
W/O SS	13.0%	12.9%	11.0%	10.3%	53%
W SS	13.6%	12.9%	11.3%	10.4%	52%
Δ	.6%		.4	0	-1%

gains share, primarily from the outside alternative. As we have seen, the other brands reduce their prices in equilibrium, compensating for the greater desirability of the Sony product.

In differentiated markets, competitive forces drive down the profits that the firm with the feature enhanced product can capture from the consumer surplus generated by the feature improvement. For this reason, pure measures of change in consumer surplus are not an appropriate measure of the value to the firm as competition will dissipate rents. To illustrate how competition effects equilibrium profits, we consider reducing the number of competitors, thus, softening competition. With product differentiation, firms will still earn positive equilibrium profits but the market power of any one firm depends not only on the number of competitors but the positions they occupy in the product space. To see this, we conduct a different market simulation in which there are only two competing firms, Sony and Canon, instead of four. The IIA property of the logit demand system at the consumer level allows us to easily compute the new market pricing equilibrium in the market with only two competitors. The IIA property means that, at the individual level, the demand system for a reduced set of alternatives (Canon, Sony, and the outside good) can be found simply by re-normalizing the choice probabilities based only the market shares for these three alternatives. We then can form market demand by integrating up over the distribution of preferences.

We find that in a system with only the Sony and Canon products, Sony faces less competition and is able to extract a greater fraction of consumer surplus from adding the swivel screen feature. The equilibrium price goes up by \$36.17 an amount larger than the change in equilibrium prices with four firms and a more densely filled in product space.

Finally, we can compute the posterior predictive distribution of the percentage change in Sony profits as the Swivel Screen feature is added. This is the ultimate measure of the value to Sony of this product feature. We can compute equilibrium profits with and without the Swivel Screen features. This computation allows other competitors to adjust their prices downward to compensate for lost demand. A more naive approach would be to assume no competitive reaction and compute the change in profits from the base equilibrium in which no firm has as Swivel Screen feature to profits earned by Sony without allowing for competitive reaction. This calculation will overstate the profit potential of the feature as it assumes that other firms will not adjust prices.

Panel (a) of Fig. 5 shows the posterior distribution of the change in Sony profits assuming no competitive reaction. The posterior mean is a 92.9 per cent increase

in profits. Panel (b) shows the results from comparisons of price equilibria with and without the feature addition. The mean change in equilibrium profit is 35.8 per cent. This represents the true economic value for the feature after accounting for competitive response and cost considerations. Given that, under our cost assumptions



Fig. 5 Posterior distribution of the change in Sony equilibrium profits

(a marginal cost of \$80 to produce the camera with SS), this translates into an increase into an incremental profit of about \$33 per unit sold.¹²

6.2 WTP computations

We have emphasized that economic valuation of product feature enhancement should be done by computing the incremental profits generated by the feature enhancement. Incremental profit calculations will take into account demand, cost, and competitive considerations. In the new products literature in economics (see, for example, Petrin (2002) and Trajtenberg (1989)), welfare based measures are typically used. That is, the new products should create additional consumer surplus and this surplus is monetarized using estimates of the marginal utility of income as in Eq. 2.11. For example, if we consider the social planner problem of how much to invest in improving trout fishing by stream improvement (see Train (1998)), then it is reasonable to use a social surplus measure to compute the social return on investment in improved recreation. However, in the private sector, the valuation of product features is determined by the firm profits that can be derived from the feature enhancement. The competitive structure of the industry will determine how the total surplus generated by the feature enhancement is divided between the consumers and the firm. However, even in the case of a monopoly, the firm profits will be less than the social surplus if the firm can only charge a uniform price. In this section, we will consider social surplus calculations as implemented via the WTP measure.

In order to compute a valid true WTP measure, we must integrate the WTP measure (shown in Eq. 2.11) over the distribution of preferences in the population. In the full Bayesian approach, we would compute the posterior predictive distribution of preferences and compute the posterior distribution of the $\mathbb{E}[\text{WTP}]$.

$$\mathbb{E}[\text{WTP}|\mu, V_\beta] = \int \text{WTP}(\beta) p(\beta|\mu, V_\beta) d\beta \quad (6.3)$$

The posterior distribution of this quantity can easily be computed using the posterior of the hyper-parameters, $p(\mu, V_\beta|Data)$. Figure 6 presents this posterior distribution. The vertical light yellow line is the mean of this distribution and a 95 percent posterior interval is shown by smaller and darker green lines.

The posterior mean of the $\mathbb{E}[\text{WTP}]$ is \$14.63. There is considerable uncertainty in this quantity but even the upper limit of a 95 per cent posterior interval is below the change in equilibrium price of about \$34. The reason for this is that the outside good share at the equilibrium prices is about 50 per cent. This means that a substantial portion of the mass of the distribution of WTP is below the market price. $\mathbb{E}[\text{WTP}]$ is, of course, an average over all potential customers

¹²The profits per unit sold are \$172-75 and 35 per cent of this is \$33.

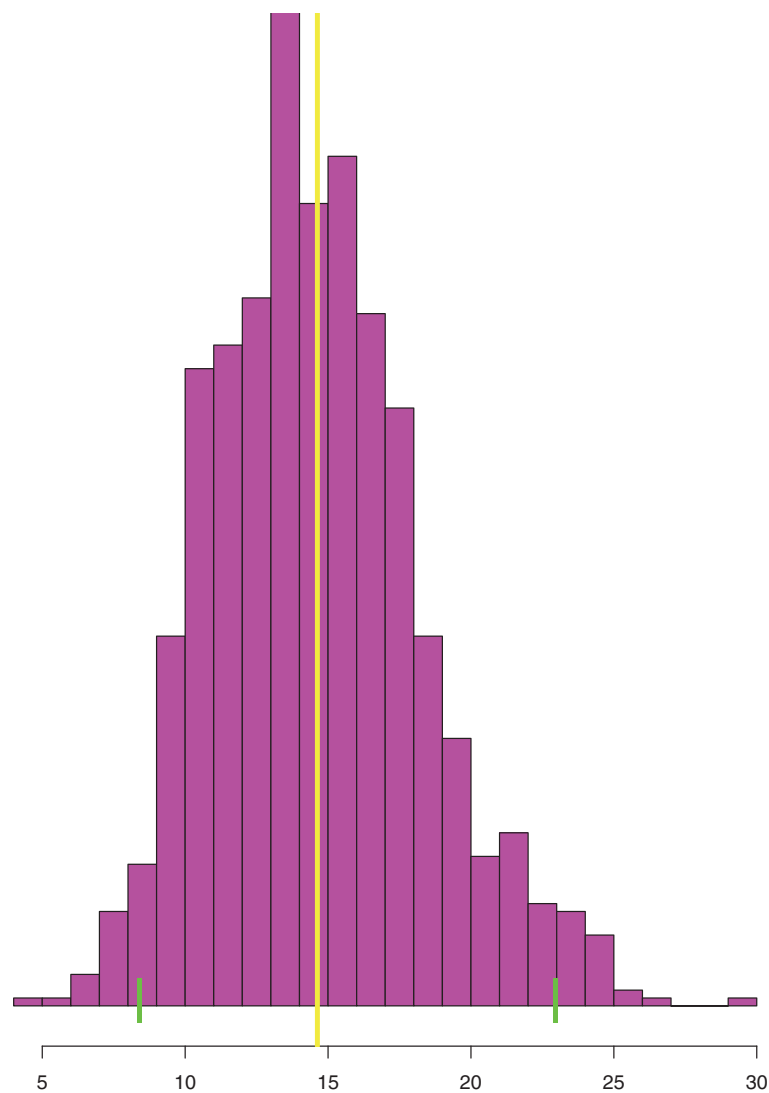


Fig. 6 Posterior Distribution of $\mathbb{E}[WTP]$

including those who would decide not to purchase a digital camera at the equilibrium prices. This is the reason why $\mathbb{E}[WTP]$ is below the change in equilibrium price.

The total social surplus generated by the feature enhancement will be the average WTP times the total size of the market. By definition, total social surplus will always exceed the firm profits attributed to the feature enhancement, since the firm can only capture part of the total consumer surplus for the subset of the market that purchases the firm's product.

Table 4 Posterior mean of aggregate demand elasticities: censored sample

Price Mkt Share	MS _{Sony}	MS _{Canon}	MS _{Nikon}	MS _{Panasonic}
P_{Sony}	−2.74	.70	.64	.70
P_{Canon}	.62	−2.48	.63	.48
P_{Nikon}	.62	.69	−2.45	.47
$P_{Panasonic}$.92	.73	.65	−2.90

6.3 Sensitivity to prior and sample selection

In conducting our analysis of the swivel screen data, we used a normal specification for heterogeneity coupled with a fairly diffuse prior. We also used all respondents except for a very small number who exhibited anomalous choice behavior such as “straight-lining.” In this section, we explore the sensitivity of our results to the choice of the sample and to the normal specification of heterogeneity. In particular, we consider removing respondents who appear to be answering more or less at random and we consider a mixture of normals specification for the distribution of preferences across respondents.

6.3.1 Sample selection

In many applications of conjoint analysis, investigators routinely do not fit the demand model on the sample of all respondents but, instead, exclude respondents who appear to be doing little more than randomly selecting product configurations. The rationale behind this censoring is that these are respondents who are not taking the survey as seriously as they would if confronted with the sample product choice task in the marketplace. These respondents could also be people who have a great deal of difficulty either understanding the conjoint choice task or in making multi-dimensional comparisons. There are many rules used to select respondents but most are based on computing model parameters at the respondent level and predicting choice for a “hold-out” sample of the conjoint choice tasks. If a respondent has a very low hit-rate (close to random choice) on the holdout sample, the respondent is excluded from analysis. There are many problems with this sort of ad hoc sample selection rule; these include choice of the size of the hold-out sample as well as the use of a crude hit rate as a index of predictability.

A more coherent approach¹³ in the same vein would be to fit the demand/choice model and compute the marginal likelihood for each respondent. If the marginal likelihood is very low, then one could argue that this respondent is choosing in a nearly random fashion. Given that there are five possible choices (four goods and the out-

¹³However, survey respondents who do not follow the compensatory model assumed by utility theory and conjoint analysis but, instead, follow some sort of screening or non-compensatory choice rule may have high likelihood. We may not be able to eliminate this type of respondent simply on the basis of in-sample fit.

Table 5 Changes in equilibrium prices: censored sample

	Sony	Canon	Nikon	Panasonic
W/O SS	\$118.49	\$126.09	\$127.45	\$114.72
W SS	\$144.09	\$124.31	\$125.24	\$114.76
Δ	\$25.60	-\$1.78	-\$2.21	-\$0.04

side option), a purely random choice would produce a marginal likelihood value of approximately $16\ln(.2) = -25.75$. We establish a cut-off for the purpose of sensitivity analysis at $16\ln(.4) = -14.66$. This cut-off eliminates 136 respondents or about 29 per cent of the original sample. Table 4 shows the new elasticity structure computed at the new vector of equilibrium prices. All elasticities are larger in magnitude than those from the full sample with a particularly sharp increase in the own price elasticity. These elasticities imply much lower equilibrium prices and lower margins.

Table 5 shows the equilibrium prices for the “baseline” and Swivel Screen enhanced product configurations. Given that these are equilibrium prices for what we have been calling the “baseline” product configurations (all non-price attributes are set to the “low” value), we find these equilibrium prices to be more reasonable and more in line with actual market prices (under our marginal cost assumption of \$75). Clearly, the censored sample is comprised of more price sensitive respondents.¹⁴

6.3.2 Non-normal heterogeneity

Virtually all industry and academic applications of choice-based conjoint use a normal distribution of heterogeneity even though the technology for handling more general flexible distributions has existed for some time. In the I/O literature, only restricted normal distributions of preferences have been used to construct market demand. The normal distribution is very flexible in terms of the pattern of covariance and variability but imposes a restricted thin-tailed and uni-modal distribution. Although we use a relatively diffuse prior, the assumption of normality itself should be viewed as a strong prior assumption. In many applications, the increased shrinkage afforded by the normal distribution is desirable. In our application, we are using the normal assumption to compute the predictive distribution of preference parameters. The predictive distribution is the ultimate source for the patterns of demand and there is a legitimate concern that the assumption of normality may strongly influence the results.

In order to assess sensitivity to the normality assumption, we re-estimated our model using a mixture of normals approach (see, for example, Rossi et al. (2005), chapter 5, Section 5). We found that if we estimated the model on the full sample of all respondents, the mixture of normals model assigned a component to very tiny

¹⁴Note that there is only a very weak correlation between time spent on the survey and the marginal log-likelihood value.



Fig. 7 Posterior predictive distribution of price and swivel screen part-worths: mixture of normals model

price coefficients and we were not able to compute equilibrium prices with a reasonable value (equilibrium prices are typically greater than \$1000 for the baseline configuration of product features). The reason for this is that this view of preferences allows for a non-negligible mass of consumers with such low price sensitivity that it is optimal for the firm to set very high prices to exploit these consumers. We do not think that this provides a realistic estimate of aggregate demand. For this reason, we estimated the mixture of normals model on the censored sample of only those respondents with appreciable log-likelihood (the same censoring criterion used in the sub-section entitled “Sample Selection” above). Figure 7 shows the distributions of the Price and Swivel Screen part-worths for a censored sample of 332 out of the original full sample of 468. The mixture of normals model produces a posterior predictive distribution for the Swivel Screen part-worth that is nearly identical to the posterior predictive from the normal model (compare to Fig. 3).¹⁵ However, the mixture normals model produces a multi-modal distribution of the price part-worth. There is a mode centered around -.5, a mode around -2.5, and a fat left tail. Comparisons to the results for the normal model (see Fig. 3), show that the normal model (as might be expected) represents a compromise between the mass of consumers with low and moderate price sensitivity.

¹⁵This is from a three component mixture of multivariate normals. A somewhat tighter prior was used with $\nu = \dim(\beta) + 25$ and the diagonal elements of V set to .5 for all part-worths except the price partworth (transformed) that has a diagonal element of .05. This prior has much thinner tails than our default settings. Without these tighter settings, the mixture of normals model will product a large enough mass of respondents with very low price sensitivity and, even with the restricted sample, we will obtain some very large equilibrium prices ($> \$500$).

Table 6 Posterior mean of aggregate demand elasticities: censored sample with a mixture of normals model

Price Mkt Share	MS_{Sony}	MS_{Canon}	MS_{Nikon}	$MS_{Panasonic}$
P_{Sony}	-2.58	.77	.05	1.07
P_{Canon}	.91	-2.70	.09	.72
P_{Nikon}	1.15	.020	-1.47	.11
$P_{Panasonic}$	1.33	.75	.05	-2.75

While the fitted distributions of price part-worths differ between the normal and mixture of normals models, it remains to be seen if this has an appreciable effect on the estimated elasticity structure for the industry and on the changes in equilibrium price from the addition of the Swivel Screen. Table 6 shows the elasticity structure for the mixture of normals model fit to the censored sample. Overall, the price elasticities are similar to those obtained from the normal model except for the case of Nikon. The mixture of normals model shows the Nikon brand with a lower own price elasticity and very small cross-elasticities with respect to the other brands. This will have profound implications for the equilibrium prices for Nikon which will be much higher in the mixture of normals specification. Examination of the implied marginals of the posterior predictive distribution of part-worths show no obvious explanation for this difference. There is a small mass of respondents with low price sensitivity and high Nikon part-worths. This mass must be driving the overall elasticity structure.

Table 7 shows equilibrium prices for the industry at the baseline product configuration and for the industry in which the Sony product is enhanced by addition of the swivel screen. The results are nearly identical to the normal results (compare to Table 5) with the exception that the Nikon equilibrium prices are higher (consistent with the lower estimated own price elasticity).

We conclude that the results of any equilibrium pricing exercise are sensitive to the assumptions regarding the form of the distribution of preferences. The applied demand literature has not yet advanced to the point of implementation of non-normal preferences but it is clear that this is a general point, not restricted to our study. Specific to our objectives, however, is a concern for a mass of respondents who appear to exhibit very small price sensitivity. Attention must turn to the question of whether it is desirable to censor the sample to remove respondents who appear to be guessing at random. Our position is that removal of this sub-sample is reasonable and will result in more realistic demand estimates.

Table 7 Changes in equilibrium prices: censored sample with a mixture of normals model

	Sony	Canon	Nikon	Panasonic
W/O SS	\$117.66	\$136.88	\$247.25	\$114.30
W SS	\$140.94	\$135.31	\$249.15	\$114.26
Δ	\$23.27	-\$1.58	-\$1.90	-\$0.04

7 Conclusions

Valuation of product features is an important part of the development and marketing of new products. We take the position that the only sensible measure of the economic value of a feature enhancement (either the addition of a completely new feature or the enhancement of an existing feature) is incremental profits. That is, we compare the equilibrium outcomes in a marketplace in which one of the products (corresponding to the focal firm) is feature enhanced with the equilibrium profits in the same marketplace but where the focal firm's product is not feature enhanced. This measure of economic value can be used to make decisions about the development of new features or to choose between a set of feature that could be enhanced.

Conjoint studies can play a vital role in feature valuation provided that they are properly designed, analyzed, and supplemented by information on the competitive and cost structure of the marketplace in which the feature-enhanced product is introduced. Conjoint methods can be used to develop a demand system but require careful attention to the inclusion of the outside option and inclusion of the relevant competing brands. Proper negativity constraints must be used to restrict the price coefficients to negative values. The Nash equilibrium prices computed on the basis of the conjoint-constructed demand system are sensitive to the precision of inference with respect to price sensitivity. This may mean larger and more informative samples than typically used in conjoint applications today. Finally, equilibrium prices can be sensitive to the assumption of a normal distribution of preferences. In our application, we find that respondents who appear to be randomly guessing provide a mass of what are estimated to be price insensitive consumers. To obtain realistic equilibrium results, we must censor the sample to remove this small but non-negligible group. The assumption of a normal preference distribution obscures identification of this group.

We illustrate our method by an application in the point and shoot digital camera market. We consider the addition of a swivel screen display to a point and shoot digital camera product. We designed a fielded a conjoint survey with all of the major brands and other major product features. Our equilibrium computations show that the economic value of the swivel screen is substantial and discernible from zero and corresponds to about a 35 per cent increase in firm profits. Social Surplus calculations which are behind WTP or equivalent measures will overstate the value of the feature enhancement as the firm captures only a fraction of the total consumer surplus generated by the feature enhancement.

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